Exact Solutions for Correlations in a S = 1/2 Spin Ladder Model having Two- and Four-Spin Exchange Interactions with Application to Inelastic Neutron Scattering

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Introduction and The Model

Following the discovery of high-T_C superconductivity in layered cuprates some two decades ago, quasi-one-dimensional spin ladders continue to attract considerable attention, since they act as a bridge or dimensional crossover between strictly one- and two-dimensional magnetic systems. Recently, it has become clearer that the prominent corrections or additions to the dominant nearest-neighbor-pair Heisenberg exchange interactions are four-spin ring (cyclic) exchange interactions around square plaquettes in the ladder model Hamiltonian.

To investigate the effects of multispin interactions in quantum spin ladders, our present studies build upon previous work that investigated a two-leg S = 1/2 spin ladder model having intra-rung two-spin and inter-rung four-spin Heisenberg exchange-type interactions and in a uniform magnetic field [1]. Due to the commutability properties of the Hamiltonian, the magnetic canonical partition function of the model was exactly calculable by a method of rung transfer matrix. More specifically, the maximum eigenvalue of the rung transfer matrix set the long-ladder limit of the system free energy per rung, enabling one to compute exact solutions for various thermodynamic quantities and ground state (T = 0) features of the ladder magnet.

Present Work: Results and Discussion

Towards a deeper understanding of the model, exact solutions are obtained for static and dynamic spin correlations [2]. The calculations are facilitated by first transforming the partition function of the S = 1/2 quantum spin ladder model having pair and quartic interactions, J_2 and J_4 respectively, and constant magnetic field into the partition function (aside from a known prefactor) of a S = 1/2 *Ising chain* having nearest-neighbor pair interactions and a *temperature-dependent* uniform magnetic field. The static correlation functions in the spin ladder model and their pertinent correlation lengths reveal a zero field ground-state phase diagram possessing a quantum critical volume, quantum critical and first-order surfaces, and a line of quantum triple points, Fig. 1**.** The dynamic correlations in the ladder magnet in zero field are used to derive an exact solution for the inelastic neutron scattering function $S(q,\omega)$ at all temperatures and arbitrary values of the interaction parameters.

Acknowledgements

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References

[1] Barry, J.H., *et al*., Phys. Rev. B, **58**, 3129-3137 (1998).
[2] Barry, J.H., *et al*., in preparation.

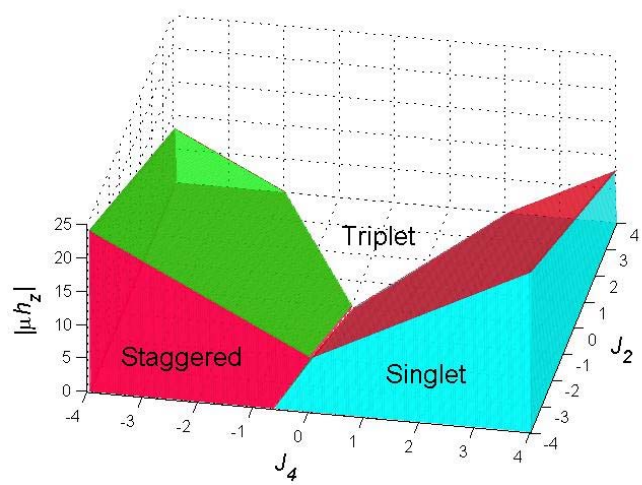

Figure. 1. Ground state phase diagram for the ladder model in terms of interaction parameters J_2 and J_4, and applied magnetic field μh_z. A quantum critical phase boundary (red plane) separates regions of rung singlets and rung triplets. The volume of staggered rungs (alternating singlets and triplets) is quantum critical in the ground state and bounded by first-order phase transitions (green plane) from the singlet and triplet states. The triplet, singlet, and staggered phases mutually coexist along a line of quantum triple-points.