Undergraduate experiment on Johnson Noise and Second Spectra of Audio Signals

Kelly L McKirahan
Colorado School of Mines, Golden, CO, 80401
Research Experience for Undergraduates 2005, National High Magnetic Field Laboratory, Florida State University, Tallahassee, FL, 32310
Mentors: Dr. Dragana Popovic, NHMFL, Tallahassee, FL, 32310
Dr. Jan Jaroszynski, NHMFL, Tallahassee, FL, 32310

Abstract

Construction of an absolute temperature Johnson noise thermometer, utilizing a resistor, is outlined and Boltzmann's constant is measured using liquid helium, liquid nitrogen, and room temperature. Construction of a converter between a 1/8 stereo-phone plug to BNC connector is outlined to measure the second spectrum of an audio input.

Introduction

Noise is a general fluctuation or change of a parameter over time. Noise is present in all electronic components and can be utilized to determine specific characteristics of materials. This electrical noise has been of increasing importance in the field of physics. Experimentation now utilizes the presence of electrical noise in components to determine the behaviors of materials. Ultimately, researchers attempt to improve the current standards of electrical components, from MOSFETs to super-conducting materials, through the understanding of molecules on a fundamental level. Within the National High Magnetic Field Laboratory experimentation is currently being done to determine the behavior of electrons within semi-conducting and super-conducting materials. Sophisticated equipment is used to measure the electronic noise produced by a small sample to determine the behavior of the electrons and ultimately characteristics of the materials. Understanding of the electron organization and orientation will allow for the improvement of current semi and super-conductors via the materials that will be utilized for construction.

The same procedure of measuring electrical noise can be altered simply to measure known constants or properties for an undergraduate research project. One of the more simplistic applications of electrical noise measurement is thermal noise generated by a resistor. Employing the Nyquist Formula one can measure kT for a given system, where k is Boltzmann's constant and T is the temperature of the system. This experiment makes use of common laboratory equipment and introduces the student to the growing field of electronic noise in physics.

Secondly, an undergraduate physics experiment was conducted to determine the second spectra of different audio signals and analyze the complexity of distinct types of signals. Determining the second spectra allows analysis of a signals discrepancy over time. Measurement of the second spectra will determine if there are patterns within natural signals and the impression of different signals upon a listener. This experimentation is meant to gain a better

understanding of audible noise and an improved knowledge of spectra and equipment used in the analysis of noise. Secondly, we wish to verify the results of M.G.A. Thomson's[1] indicating that natural signals demonstrate non-Gaussian behavior and the greater complexity of the signal the larger the impression upon the listener.

Background

Johnson-Nyquist noise was discovered in 1927 and published in the Physical Review by Johnson; the theory was completed in 1928 by Nyquist. Together they determined that a resistor thermally generated the mean square voltage $<V^2>$ given by:

$$d<V^2> = 4kTRd\mathit{f} \qquad [1]$$

In this equation R is the resistance of the resistor, T is the temperature of the system, and f is the frequency. This equation may be rewritten to determine the input voltage to the amplifier as:

$$<V_i^2> = 4kTR\Delta\mathit{f} \qquad [2]$$

For a simple lowpass filter $\Delta\mathit{f} = (\pi/2)$ which is the bandwidth of the amplifier.

Equipment

Part 1: Johnson Noise

One difficulty in measuring electronic noise is the superfluous noise generated by the equipment used in the experiment and subsequent background noise that skew the results. This added noise can either be eliminated or measured as background noise and then subtracted from the total noise present to determine the Johnson noise. As the first option can be rather difficult to achieve the second method was employed to remove any extraneous noise.

Amplifier noise performance is specified by the amount of noise the amplifier adds to the amplified thermal noise and is referred to as the noise figure, NF, and expressed in decibels.

$$NF = 10 \log_{10} (<V_i^2>/ 4kTR\Delta\mathit{f}) \qquad [3]$$

Every amplifier has specific NF characteristics that are dependant upon frequency and source resistance. The NF of the low-noise amplifier is generally located in the user manual as a contour plot at 290K in the frequency resistance plane for constant NF. Figure 1 depicts the NF contour plot provided from the manufacturer for the EG&G Princeton Applied Research Model 124A lock-in amplifier that filtered the pass band frequency in the Johnson noise experiment. To minimize the NF of the system there is an optimum frequency defined to be $\delta NF / \delta\mathit{f} = 0$. However, it is easier to calculate the optimum source for the different frequencies defined to be, R_{opt}.

$$R_{opt} = (<V_n^2> / <I_n^2>)^{1/2} \qquad [4]$$

In this experiment to decrease the noise figure of the experimentation a 10 kΩ resistor at 1 kHz generating a 0.5 dB noise figure according to Figure 1.

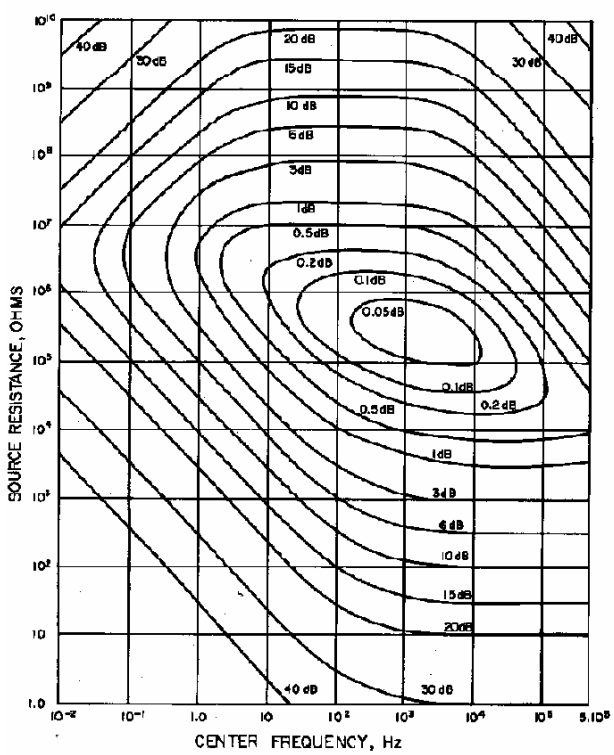

FIG. 1. Typical noise figure contour plot for the PAR116 depicting the operating parameters of the lock-in amplifier.[2]

The source thermal noise, which contributes to the total noise, is dependant on bandwidth. The bandwidth can be reduced by setting the amplifier on Bandpass mode and setting the Q switch to 10% equivalent noise bandwidth, ENBW. In the case of a 10 kΩ resistor at 1 kHz equation [1] holds true. It should be noted that exceeding the shunting capacitance, noted in the users manual for each individual amplifier, causes the capacitance to change the ENBW from equation [1] to:

$$\Delta`f = \text{Integrate}[\text{Abs}[G(f)]^2]\ df/\ G_0^2 * (1+f^2/f_0^2) \quad [5]$$

It should be noted that ENBW is a model that pertains to wideband noise. "The concept of equivalent noise bandwidth arises from the fact that Noise Bandwidth is an unattenuated rectangular bandwidth, while signal bandwidths are specified as the number of Hz between two points of given attenuation on the response characteristics…The equivalent noise bandwidth of the filter is defined as the unattenuated bandwidth of an equivalent theoretical (but physically impossible) perfectly sharp filter that, with the same wideband input noise, yields the same rms output noise amplitude as the filter of interest."[2] Figure 2 indicates the bandpass characteristics of the PAR-116 from which we derive the frequency bandwidth of 1 kHz for equation [1] as our optimal frequency with a 10 kΩ resistor and a low noise figure of 0.5 dB.

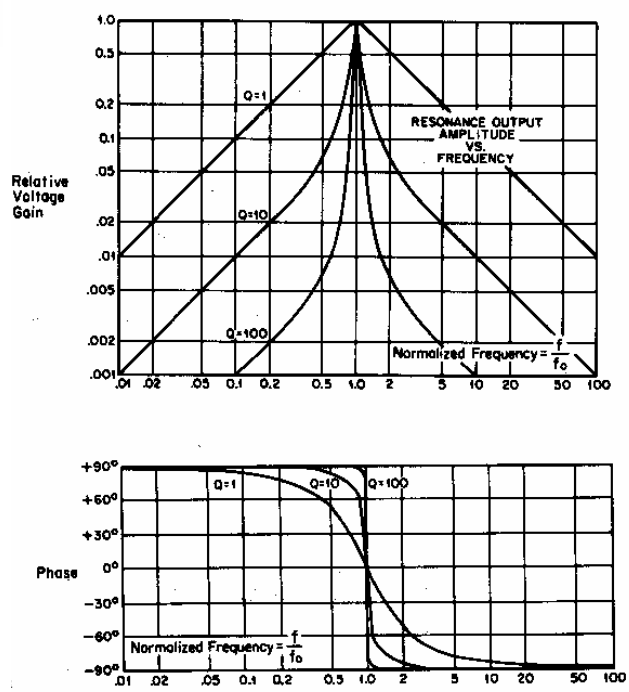

FIG. 2. Typical bandpass characteristics for the PAR-116 lock-in amplifier. In this experiment Q was set equal to 10% to accommodate equation [1]. [2]

Note that the frequency bandwidth, Δf , can be determined

through measurement of the output signal from the lock-in amplifier.

Δf = Area beneath the output signal/ Y-max [6]

The peak of the output signal is designated Y-max. Measurement of Δf will decrease the error within the experimental results. The ENBW was measured directly in the lab using a HP35665A Dynamic Signal Analyzer and Δf was directly measured for use in calculating Boltzmann's constant.

When choosing a resistor for use in a Johnson Noise experiment it is best to consider resistors that do not vary significantly at varying temperatures. Carbon resistors, which are very common in laboratories, are temperature dependant generating "excess noise". Due to afore mentioned characteristics, carbon resistors are generally ill suited for noise thermometry. Alternatives to the carbon resistors are the wire wound resistor or the metal film resistor which are optimal at a wide variety of temperatures. These two types of resistors are well suited to different temperatures because their resistivity is relatively unaffected by changes in temperature. In this experiment the metal film resistor was utilized for testing Johnson noise due to availability.

When choosing a voltmeter to measure the output of the PAR-116 Lock-in it is best to chose a lock-in that reacts to the mean square voltage of the wave form or rms. As is common, if a voltmeter reacts to the mean of the half-wave rectified signal a correction factor will be needed to convert the data to rms.

To avoid excess noise it is necessary to ensure that the experiment has correct grounding. The generation of large, emf induced, current from ground loops, various paths to ground current, will increase the noise measured. In such low-noise measurements it is especially important to ensure no ground loops exist.

Part 2: Second Spectra of Audio Signals

This experiment utilized a Dynamic Signal Analyzer (DSA) HP35665A to measure different audio signals. A DSA is ideal for the measurement of noise due to its ability to analyze over various frequency ranges. When selecting an analyzer to measure audio signals ensure that the analyzer has the capability to read signals throughout the entire audio range.

A computer interface for the analyzer is very helpful in analysis of results. If the equipment provided is unequipped with an interface, such as LabVIEW, the simplest solution is to contact the manufactures who will generally provide drivers online or by request for their products.

One method of measuring audio signals using a DSA is to construct a 1/8`` stereo-phone plug to BNC converter to transfer an audio signal from a computer to the analyzer this setup depicted in figure 3.

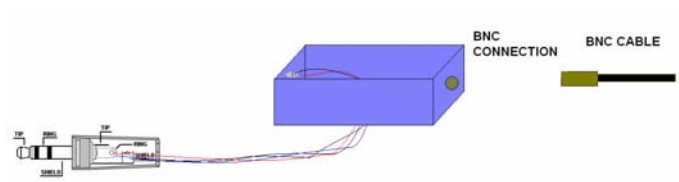

FIG. 3. Diagram of a 1/8`` stereo-phone plug to BNC cable converter.

When measuring the spectrum for audible noise it is important to observe

correct grounding and shielding techniques as noted in part 1.

Experimental Setup

Part 1: Johnson Noise

A precision metal film resistor (10 kΩ) was used to reduce unwanted noise and because of the relatively low dependence of the resistivity on temperature. The resistor was placed in a stainless steel probe approximately 50`` long that was grounded and shielded. The probe is constructed of stainless steel due to it's the fact that it is a poor conductor of heat. Ultimately this correlates to less liquid helium boiling off. The resistor is located at the end of the probe within a closed section constructed of brass. Before sealing the probe the BNC outputs were checked to ensure that the resistance between the two outputs was 10 kΩ, or the resistance of your chosen resistor. These two BNC outputs will be connected to the lock-in amplifier inputs A and B. To reduce the amount of noise generated the cables were twisted together between the probe connection and the lock-in amplifier.

The lock-in amplifier used in this experiment was the PAR-116 in Bandpass mode with Q set to 10% and a frequency of 1000Hz. To decrease the amount of extraneous noise all unnecessary equipment was turned off during experimentation.

The noise of the resistor was measured in the probe at room temperature, which was approximately 294K. The probe was then cooled and slowly lowered into a liquid nitrogen bath, of approximately 77K. It is important to allow the probe to come to thermal equilibrium with the liquid nitrogen before beginning any measurements. To ensure good results allow the probe to remain in the bath until the lock-in amplifier has reached a steady, constant output. Lastly, the probe was slowly lowered into a liquid helium dewer, of approximately 4.2K. When using liquid helium it is important to lower the probe very slowly so as to reduce the amount of helium that boils as it comes into contact with the warmer surface of the stainless steel probe. The slow process of lowering the probe should allow the probe to come to thermal equilibrium with the liquid helium rather slowly.

Part 2: Second Spectra of Audio Signals

Firstly, it is important to create an interface between the computer and the HP35665A DSA used for the collection of data. Unfortunately, due to the age of the hardware and the subsequent release of newer DSA models, Hewlett-Packard does not provide a pre-made interface or drivers for the analyzer I used. Therefore, construction of the HP35665A needed to be done in lab. The solution arrived at was to use drivers for the newer model of DSA produced by Hewlett-Packard and Agilent Technologies. LabVIEW is one of the most widely used interfacing software programs. LabVIEW 6.1 was therefore chosen for the construction of the interface. National Instruments provides a networking and customer service webpage that provides a compilation of drivers built for a number of types of common lab equipment. Note that using drivers for more recent equipment is not always a possible or good option. When building an interface it is important to know how the interface will be designed to operate. Will the user

need to have access to the entire front panel of the hardware or just a few specific functions? Is an interface necessary or could the machine be run using a simple VISA read/write command VI? Who is the ultimate user of the interface? Does this interface need to be relatively user friendly? How much time will be allotted for the construction of an interface? How well acquainted with LabVIEW or another interface program are you? Building a LabVIEW interface and drivers from scratch can be a very difficult and time consuming task for a beginner. If at all possible drivers should be secured from a third party. If you decide to construct a LabVIEW program using newer drivers ensure that the newer equipment has the same functions that you will need for your hardware. When using drivers created for a different machine be prepared to improvise and expect errors. Remember that if you must choose drivers that are not specifically for your machine ensure that they are from the same manufacturer. If the machines have the same manufacturer they have a greater chance of compatibility and fewer errors.

Connecting the computer to channel 1 of the HP35665A, using the converter depicted in figure 3, enables the user to view the power spectrum, the distribution of the signal as a function of frequency, of different input signals. In this experiment classical music, rock music, and speech were input to the DSA. The computer interface using LabVIEW 6.1 collected a series of power spectra between 4 and 10 minutes in duration. Samples were taken from Charles Munch, Gustav Mahler, Pink Floyd, Weezer, and a MSNBC news broadcast. A Mathematica program was then used to extract the second spectrum from the collected data.

Results

Part 1: Johnson Noise

Figure 4 depicts the measured ENBW using the HP35665A and a computer interface with LabVIEW 6.1. Using equation [6] we are able to measure the actual Δf of the experiment to be 1.15 kHz rather than 1 kHz as selected on the PAR-116.

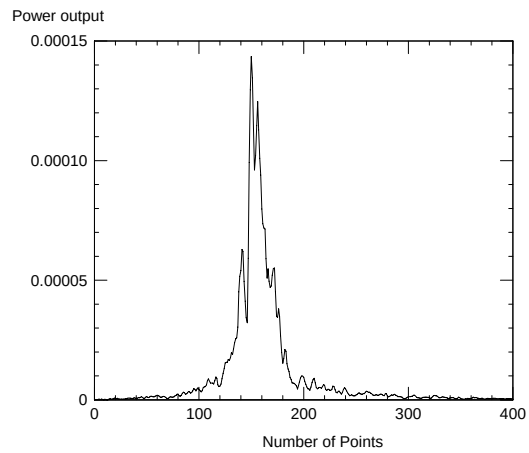

FIG. 4. HP35665A average of 100 power spectrum of the ENBW. Experimental measurement of the signal from the PAR-116 determined that in actuality Δf is equal to 1.15 kHz.

The Johnson-Nyquist noise was measured at three temperatures: 300K, 77K, and 4.2K. For each of these temperatures equation [2] was used to calculate Boltzmann's constant. Figure 5 shows the experimental results of voltage ($d\langle V^2\rangle$) versus temperature (K). Using the slope of the fit line and equation [2] Boltzmann's constant was experimentally found to be 1.88E-23 J/K. It should be noted that large error occurred in the measurement because of the low number of data sets, the error involved in measuring small quantities such as nano-volts, and the presence of background noise during the

experimentation. The experimental value of k is in reasonably good agreement with the established value of k, 1.381E-23 J/K.

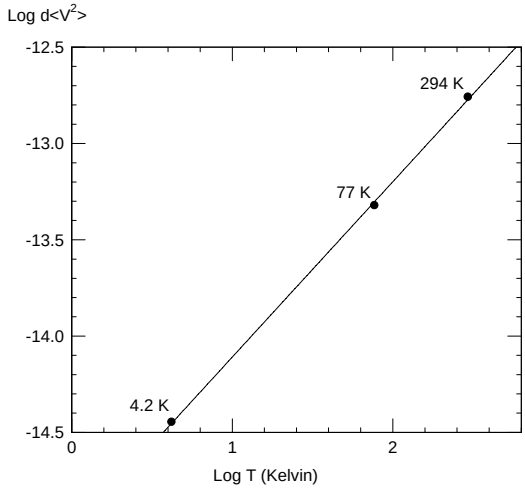

FIG. 5. Noise thermometer plot at three different temperatures.

Part 2: Second Spectra of Audio Signals

Experimental results concluded that classical music had a more complicated second spectrum than rock music or speech. Speech tended to have a “white” second spectrum, while rock generally had a “pink” spectrum in the lower frequencies with a whiter spectrum in the higher frequencies. The classical music tended to have a pink = 1/f spectrum. These results for music and speech are verified within M.G.A. Thomson’s “Sensory Coding and the second Spectra of Natural Signals,” that asserts the greater the complexity of the second spectra the greater the impression upon the listener. Figure 6 shows the different second spectra of classical, rock, and speech. These spectra have large scattering due to the limited amount of time over which the samples were collected, 4 to 10 minutes.

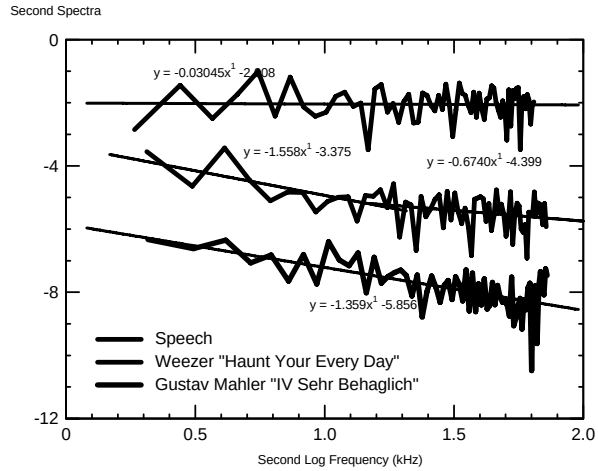

FIG. 6. The second spectrum of a selection of three audio signals: speech, Weezer, and Gustav Mahler.

Resources

1 M.G.A. Thomson. “Sensory Coding and the Second Spectra of Natural Signals,” Physical Review Letters, Volume 86, Number 13, 2901. The American Physical Society, 2001.

2 “Model 124A Lock-In Amplifier: Operating and Service Manual,” EG&G Princeton Applied Research,1983.