

Pixelation Project: Creating Optical Illusions



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Pixel Project

Rational: The 3-5th grade student will understand the relationship between the human visual system and technology of the 21st century.

Goal: Students will learn about how the eye works to form optical illusions and relate this knowledge to have a better understanding of how humans interact with technology.

Grade Level: 3-5th

Subject: Science and Math

Lesson Objective:

Math: Given a fraction, the 3rd-5th grade student will color a representation of the fraction value, to demonstrate understanding of fractions.

Science: Upon completion of the fraction squares, the 3rd- 5th grade student will combine the squares to develop a pixelized picture to demonstrate understanding of grayscale and pixels.

FL. Sunshine State Standards:

Science:

SC.H.1.2.1. Knows that a model of something is different from the real thing, but can be used to learn something about the real thing.

SC.H.3.2.1. Understands that people, alone or in groups, invent new tools to solve problems and do work that affects aspects of life outside of science.

SC.H.3.2.2. Knows that data are collected and interpreted in order to explain an event or concept.

SC.H.3.2.3. Knows that before a group of people build something or try something new, they should determine how it may affect other people.

SC.H.3.2.4. Knows that through the use of science processes and knowledge, people can solve problems, make decisions, and form new ideas.

Math:

MA.A.1.2.1. Names whole numbers combining three-digit numeration (hundreds, tens, ones) and the use of number periods, such as ones, thousands, and millions and associates verbal names, written word names, and standard numerals with whole numbers, commonly used fractions, decimals, and percents.

MA.A.1.2.2. Understands the relative size of whole numbers, commonly used fractions, decimals, and percents.

MA.A.1.2.3. Understands concrete and symbolic representations of whole numbers, fractions, decimals, and percents in real-world situations.

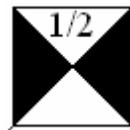
MA.A.1.2.4. Understands that numbers can be represented in a variety of equivalent forms using whole numbers, decimals, fractions, and percents.

Materials:

- Black permanent markers
- Scissors
- Rulers
- Pencils
- Black markers
- Tape

Procedure:

1. Review Teacher Background (**Appendix A**)
2. Student Handout Available (**Appendix B**)
3. Remove the Fraction sheets (**Appendix C**)
4. Give each student a numbered page of the document.
5. Instruct the students to use a pencil and black marker to outline the fraction value of each cell. Example:



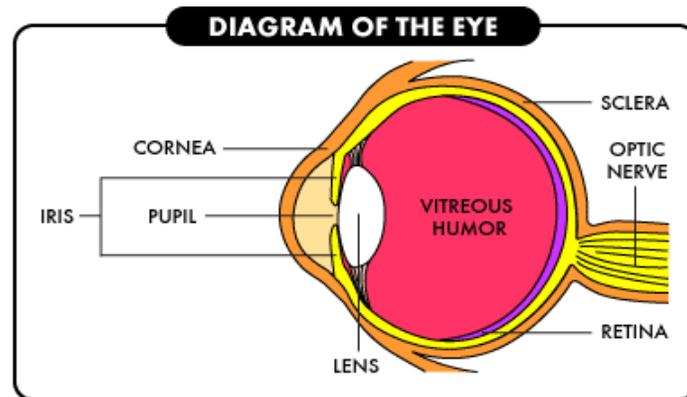
6. Trim the edges of the pages so that they easily fit together.
7. Tape each page using the Page Map (**Appendix D**)
8. Have the students stand close to the picture to see if they see a clear image.
9. Have the students stand back 30-40 feet to see if there is a difference.

Appendix A

Teacher Background

Background:

- Whenever you see an object it's because light rays are reflected through the lens in your eye. This lens is so flexible that you can focus on near and far objects and you can see things that you're not even focusing on.
- The light rays pass through the jelly-like eyeball and onto the retina- a screen in the back of your eye. The retina is made up of many special cells that are sensitive to light and color. From the retina, visual messages are relayed to the brain along a pathway called the optic nerve. Your brain then interprets this information into a picture.



- Today's modern televisions, computer monitors, and digital cameras utilize a similar process by storing images into points or pixels. Each pixel has its own color information and brightness. The more pixels in the picture, the more detailed the image appears.
- The eye's retina contains receptors in the back. All of the receptors are assimilated into a single image, because our brains are so sophisticated that they can process this information all at once. However this same sophistication is what tricks us every day.
- Televisions and computer screens are broken down into pixels. These pixels are continually changing. This is why we see the pictures as real time moving objects.
- Pixels start at the upper left hand corner of a screen. They add on to the right at a rapid speed. Each pixel is either a color or a grey-scale value. The pixels continue to refresh.
- Our eye is tricked into thinking that this is one picture and not a series of pixels.

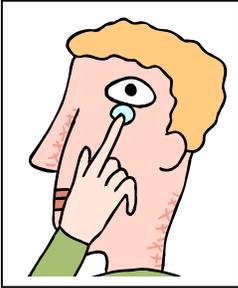
Appendix B

Student Worksheet

(Pixelation and You)

Pixelization and You

Whenever you see an object it's because light rays are reflected through the lens in your eye. This lens is so flexible that you can focus on near and far objects and you can see things that you're not even focusing on.



In the back your eye you have a screen like area called the retina. Light rays pass through your jelly-like eyeball and project onto your retina. Your brain then interprets this information into a picture.

Your HDTV, computer monitors, and digital cameras work in a very similar way. They all store images into points or pixels. Each pixel has its own color information and brightness. The more pixels in the picture, the more detailed the image appears.

Your eye's retina contains many receptors cells in the back. These receptor cells are like pixels on the screen.



The television, camera and monitor are broken down into pixels. These pixels are always changing. This is why we see the pictures moving objects. Pixels start at the upper left hand corner of a screen. They add on to the right at a rapid speed. Each pixel is either a color or a grey-scale value. The pixels continue to refresh. You eye is tricked into thinking that this is one picture and not a series of pixels. In other words, your eyes blend the squares to form one image.

Appendix C

Fraction Worksheets

0 (page 1)	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0 (page 2)	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0 (page 3)	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0 (page 4)	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0 (page 5)	0	0	0
0	0	0	0
0	0	0	1/6
0	0	1/9	3/4

0(page 6)	0	$\frac{2}{3}$	$\frac{5}{7}$
0	$\frac{1}{7}$	$\frac{6}{7}$	$\frac{5}{7}$
0	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{4}{5}$
0	$\frac{2}{7}$	$\frac{5}{8}$	$\frac{2}{3}$

0 (page 7)	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{3}{5}$
0	$\frac{2}{7}$	$\frac{3}{5}$	$\frac{3}{5}$
0	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{3}{5}$
0	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{3}{5}$

1/6(page 8)	2/3	3/5	3/5
1/4	3/5	3/5	3/5
1/3	3/5	3/5	3/5
1/3	3/5	3/5	2/3

$2/7$ (page 9)	$5/8$	$3/5$	$2/3$
0	$2/7$	$2/7$	$1/2$
0	0	0	$1/9$
1	1	1	1

0(page 10)	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0 (page 11)	0	0	0
0	0	0	4/9
0	0	1/7	7/9
0	0	4/7	2/3

0 (page 12)	$\frac{1}{9}$	$\frac{4}{5}$	$\frac{2}{3}$
0	$\frac{2}{5}$	$\frac{3}{4}$	$\frac{2}{3}$
0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
0	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{2}{3}$

1/9 (page 13)	4/5	2/3	2/3
2/9	6/7	5/7	5/7
1/4	7/9	2/3	2/3
1/4	4/5	5/7	5/7

$2/9$ (page 14)	$5/7$	$3/5$	$3/5$
$2/9$	$4/5$	$2/3$	$2/3$
$2/3$	$4/5$	$6/7$	$4/7$
$5/7$	$5/7$	$3/4$	$7/8$

$5/7$ (page 15)	$5/7$	$5/7$	$5/6$
$5/7$	$5/7$	$5/7$	$3/4$
$3/4$	$5/7$	$5/7$	$5/7$
$2/3$	$2/3$	$2/3$	$2/3$

$\frac{3}{5}$ (page 16)	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{2}{3}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{5}{7}$

$\frac{3}{5}$ (page 17)	$\frac{4}{5}$	$\frac{5}{8}$	$\frac{3}{5}$
$\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{7}{9}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{3}{4}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{3}{5}$

$3/4$ (page 18)	$7/9$	$5/7$	$3/4$
$5/6$	$5/7$	$5/7$	$4/5$
$6/7$	$3/4$	$5/7$	$4/5$
1	1	1	1

0 (page 19)	0	0	0
0	0	0	0
0	0	0	0
0	$\frac{1}{4}$	$\frac{3}{5}$	$\frac{5}{8}$

$\frac{1}{2}$ (page 20)	$\frac{7}{9}$	$\frac{6}{7}$	$\frac{2}{3}$
$\frac{5}{7}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{5}{6}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{5}{6}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{4}$

$\frac{2}{3}$ (page 21)	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{5}{6}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{6}{7}$	$\frac{4}{5}$
$\frac{2}{3}$	$\frac{7}{9}$	$\frac{5}{7}$	$\frac{5}{8}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{5}$

$\frac{2}{3}$ (page 22)	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{7}{8}$
$\frac{5}{7}$	$\frac{5}{7}$	$\frac{5}{7}$	$\frac{4}{5}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

$\frac{3}{5}$ (Page 23)	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{5}{7}$	$\frac{5}{7}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

5/7 (Page 24)	5/8	2/3	5/8
4/5	2/3	2/3	2/3
6/7	5/7	2/3	2/3
5/6	3/4	2/3	2/3

$\frac{2}{3}$ (page 25)	$\frac{6}{7}$	$\frac{3}{5}$	$\frac{4}{7}$
$\frac{3}{5}$	$\frac{4}{5}$	$\frac{7}{9}$	$\frac{5}{7}$
$\frac{2}{3}$	$\frac{3}{4}$	1	$\frac{2}{3}$
$\frac{2}{3}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{7}{9}$

$\frac{3}{5}$ (Page 26)	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{5}{8}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$

$\frac{3}{4}$ (Page 27)	$\frac{5}{7}$	$\frac{4}{5}$	$\frac{2}{3}$
$\frac{4}{5}$	$\frac{5}{7}$	$\frac{7}{9}$	$\frac{5}{7}$
$\frac{4}{5}$	$\frac{5}{7}$	$\frac{3}{4}$	$\frac{3}{4}$
1	1	1	1

0(Page 28)	0	0	0
0	0	0	0
0	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{3}{8}$
$\frac{4}{5}$	$\frac{5}{7}$	$\frac{2}{3}$	$\frac{4}{5}$

$\frac{4}{5}$ (Page 29)	$\frac{4}{7}$	$\frac{4}{5}$	$\frac{3}{4}$
$\frac{4}{5}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{5}{7}$
$\frac{7}{9}$	$\frac{2}{3}$	$\frac{7}{9}$	$\frac{2}{3}$
$\frac{7}{9}$	$\frac{3}{4}$	$\frac{5}{7}$	$\frac{5}{7}$

$\frac{2}{3}$ (Page 30)	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{5}{7}$	$\frac{5}{7}$	$\frac{3}{4}$	$\frac{2}{3}$
$\frac{5}{7}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{3}{4}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$

$3/5$ (Page 31)	$3/5$	$3/5$	$2/3$
$5/7$	$3/5$	$3/5$	$3/5$
$6/7$	$5/8$	$5/8$	$2/3$
$5/7$	$5/6$	$3/5$	$3/5$

$\frac{2}{3}$ (Page 32)	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{5}{7}$
$\frac{5}{7}$	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{3}{5}$
$\frac{2}{9}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{7}{9}$
$\frac{2}{3}$	$\frac{4}{5}$	$\frac{7}{9}$	$\frac{5}{7}$

$\frac{3}{5}$ (Page 33)	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{3}{4}$
$\frac{5}{7}$	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{7}$
$\frac{2}{3}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{7}{9}$

4/7 (Page 34)	2/3	3/4	2/3
2/3	7/9	2/3	3/5
2/3	5/7	3/4	3/5
4/5	3/4	3/4	3/5

$\frac{2}{3}$ (Page 35)	$\frac{6}{7}$	$\frac{5}{8}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$
$\frac{5}{8}$	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{3}{5}$
$\frac{4}{5}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{5}$

$\frac{3}{4}$ (Page 36)	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{5}{8}$
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{1}{3}$
$\frac{4}{5}$	$\frac{5}{7}$	1	$\frac{1}{4}$
1	1	1	1

$\frac{1}{8}$ (Page 37)	$\frac{7}{9}$	$\frac{3}{7}$	0
$\frac{1}{4}$	$\frac{7}{9}$	$\frac{4}{7}$	0
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{5}$
$\frac{3}{4}$	$\frac{2}{3}$	$\frac{7}{9}$	$\frac{4}{5}$

$\frac{2}{3}$ (Page 38)	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{7}{9}$
$\frac{5}{8}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{7}{9}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

$\frac{3}{5}$ (Page 39)	$\frac{3}{5}$	$\frac{5}{8}$	$\frac{2}{3}$
$\frac{5}{7}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$
$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$

$5/7$ (Page 40)	$5/7$	$5/7$	$3/4$
$5/6$	$3/4$	$3/4$	$5/7$
$4/5$	$1/2$	$1/2$	$8/9$
$6/7$	$2/3$	$2/3$	$6/7$

$\frac{7}{9}$ (Page 41)	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{6}$
$\frac{4}{5}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{7}{8}$
$\frac{3}{4}$	$\frac{7}{9}$	1	$\frac{5}{7}$
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{8}{9}$	$\frac{3}{4}$

$\frac{3}{4}$ (Page 42)	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{4}{5}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{5}{8}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{2}{3}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{7}$	$\frac{2}{3}$

$\frac{3}{5}$ (Page 43)	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{8}$	$\frac{3}{4}$

$\frac{3}{5}$ (Page 44)	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{4}{5}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
0	0	0	0
1	1	1	1

0(Page 45)	0	0	0
0	0	0	0
$\frac{2}{7}$	$\frac{1}{5}$	0	0
$\frac{1}{2}$	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{2}{3}$

$\frac{1}{2}$ (Page 46)	$\frac{7}{9}$	$\frac{5}{8}$	$\frac{6}{7}$
$\frac{3}{5}$	$\frac{3}{4}$	$\frac{5}{7}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{2}{3}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{8}{9}$	$\frac{2}{3}$

$\frac{2}{3}$ (Page 47)	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{7}{9}$
$\frac{5}{7}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{4}{7}$	$\frac{5}{8}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{8}$	$\frac{2}{3}$

$\frac{3}{5}$ (Page 48)	$\frac{3}{5}$	$\frac{8}{9}$	$\frac{4}{5}$
$\frac{3}{5}$	$\frac{5}{7}$	$\frac{3}{4}$	$\frac{2}{3}$
$\frac{3}{5}$	$\frac{3}{4}$	$\frac{4}{7}$	$\frac{4}{7}$
$\frac{2}{3}$	1	$\frac{3}{4}$	$\frac{5}{7}$

$5/7$ (Page 49)	$5/7$	$4/5$	$2/3$
$2/3$	$7/9$	$4/5$	$2/3$
$7/9$	$3/4$	$5/6$	$2/3$
$5/7$	$5/7$	$4/5$	$2/3$

$5/7$ (Page 50)	$5/7$	$5/6$	$3/4$
$5/7$	$5/7$	$3/4$	$2/3$
$5/7$	$5/7$	$4/5$	$2/3$
$7/9$	$3/4$	$5/6$	$5/7$

$5/7$ (Page 51)	$5/7$	$5/6$	$2/3$
$3/5$	$3/5$	$4/5$	$2/3$
$3/5$	$3/5$	$4/5$	$2/3$
$3/5$	$3/5$	$5/6$	$7/9$

$\frac{3}{5}$ (Page 52)	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{5}{8}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{4}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{6}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{4}{5}$

$\frac{3}{5}$ (Page 53)	$\frac{2}{3}$	$\frac{5}{7}$	$\frac{3}{4}$
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{3}{4}$
0	0	0	$\frac{2}{3}$
1	1	1	1

0(Page 54)	0	0	0
0	0	0	0
0	0	0	0
2/9	0	0	0

$\frac{7}{9}$ (Page 55)	$\frac{1}{3}$	0	0
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{7}$	0
$\frac{2}{3}$	$\frac{5}{7}$	$\frac{4}{5}$	$\frac{1}{9}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{5}$

$\frac{3}{4}$ (Page 56)	$\frac{2}{3}$	$\frac{5}{7}$	$\frac{5}{7}$
$\frac{5}{7}$	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{5}$
$\frac{3}{5}$	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{2}{3}$

$\frac{2}{3}$ (Page 57)	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{2}{33}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{4}{7}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{5}{7}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

$\frac{2}{3}$ (Page 58)	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{5}{6}$
$\frac{2}{3}$	$\frac{5}{7}$	$\frac{5}{7}$	$\frac{5}{6}$

$5/7$ (Page 59)	$5/7$	$5/7$	$5/6$
$5/8$	$3/5$	$2/3$	$4/5$
$2/3$	$2/3$	$2/3$	$6/7$
$5/7$	$2/3$	$2/3$	$4/5$

$\frac{2}{3}$ (Page 60)	$\frac{2}{3}$	$\frac{5}{7}$	$\frac{5}{7}$
$\frac{2}{3}$	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{5}{8}$
$\frac{5}{7}$	$\frac{7}{9}$	$\frac{7}{9}$	$\frac{2}{3}$
$\frac{4}{5}$	$\frac{8}{9}$	$\frac{2}{3}$	$\frac{3}{5}$

7/9(Page 61)	5/8	3/5	3/5
2/3	3/5	3/5	3/5
3/5	2/3	3/5	5/8
4/5	4/5	5/8	4/5

$\frac{6}{7}$ (Page 62)	$\frac{5}{7}$	$\frac{3}{4}$	$\frac{4}{5}$
$\frac{4}{5}$	$\frac{5}{7}$	$\frac{3}{4}$	$\frac{7}{9}$
$\frac{3}{4}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{7}{9}$
1	1	1	1

0(Page 63)	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0(Page 64)	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0(Page 65)	0	0	0
$\frac{1}{6}$	0	0	0
$\frac{3}{7}$	0	0	0
$\frac{1}{2}$	0	0	0

5/8(Page 66)	0	0	0
2/3	0	0	0
3/5	0	0	0
3/4	0	0	0

$\frac{2}{3}$ (Page 67)	$\frac{1}{2}$	0	0
$\frac{6}{7}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{6}$
$\frac{5}{7}$	$\frac{5}{7}$	$\frac{5}{7}$	$\frac{5}{6}$
$\frac{5}{7}$	$\frac{5}{7}$	$\frac{3}{4}$	$\frac{2}{3}$

$5/7$ (Page 68)	$5/7$	$5/7$	$5/9$
$5/7$	$4/5$	$4/7$	$7/9$
$3/4$	$3/5$	$3/5$	$2/3$
$3/5$	$3/5$	$3/5$	$3/5$

$\frac{3}{5}$ (Page 69)	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{2}{3}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{7}$	$\frac{3}{4}$

$\frac{3}{5}$ (Page 70)	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{2}{3}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{5}{7}$
$\frac{5}{7}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{5}{8}$

$5/7$ (Page 71)	$3/4$	$2/3$	$4/5$
$5/7$	$5/7$	$3/4$	$4/5$
$5/7$	$3/4$	$7/9$	$5/7$
1	1	1	1

0(Page 72)	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0(Page 73)	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0(Page 74)	0	0	0
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0(Page 75)	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0(Page 76)	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

$\frac{5}{6}$ (Page 77)	$\frac{1}{3}$	0	0
$\frac{5}{8}$	$\frac{1}{2}$	0	0
$\frac{3}{5}$	$\frac{5}{8}$	0	0
$\frac{3}{5}$	$\frac{2}{3}$	0	0

$\frac{3}{5}$ (Page 78)	$\frac{3}{4}$	$\frac{1}{8}$	0
$\frac{3}{5}$	$\frac{5}{8}$	$\frac{1}{2}$	0
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{1}{9}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{1}{2}$

$\frac{3}{4}$ (Page 79)	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{7}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{2}{3}$	$\frac{7}{9}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{4}{5}$	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{3}{5}$

$5/7$ (Page 80)	$5/6$	$2/3$	$1/2$
$5/7$	$5/6$	$2/5$	0
$5/9$	$4/9$	$1/9$	0
1	1	1	1

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1/6(Page 89)

3/5

4/5

5/6

2/5(Page 90)

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Appendix D

Page Map

1	10	19	28	37	46	55	64	73	82
2	11	20	29	38	47	56	65	74	83
3	12	21	30	39	48	57	66	75	84
4	13	22	31	40	49	58	67	76	85
5	14	23	32	41	50	59	68	77	86
6	15	24	33	42	51	60	69	78	87
7	16	25	34	43	52	61	70	79	88
8	17	26	35	44	53	62	71	80	89
9	18	27	36	45	54	63	72	81	90

